

SECTION-A

- 1) $\tan(2 \tan^{-1} \frac{1}{5}) = \tan\left[\tan^{-1}\left(\frac{2 \times \frac{1}{5}}{1 - (\frac{1}{5})^2}\right)\right] = \tan\left(\tan^{-1}\left(\frac{2/5}{24/25}\right)\right) = \frac{5}{12} //$
- 2) $f \circ g\left(-\frac{5}{2}\right) = f\left(g\left(-\frac{5}{2}\right)\right) = f\left(-\frac{5}{2}\right) = \left[\frac{5}{2}\right] = \underline{\underline{2}}$
- 3) $\sin\left[\frac{\pi}{3} + \sin^{-1}\frac{1}{2}\right] = \sin\left[\frac{\pi}{3} + \frac{\pi}{6}\right] = \sin\frac{\pi}{2} = 1$
- 4) $-2x \sin x^2 \cdot \cos(\cos x^2)$

SECTION-B

- 5) $2*(x+5-5x) = 13 \Rightarrow 2*(5-4x) = 13$
 $\Rightarrow 2+5-4x-10+8x = 13 \Rightarrow 4x-3 = 13 \Rightarrow 4x = 16 \Rightarrow \underline{\underline{x = 4}}$
- 6) $y = \cos x \cdot \cos 2x \cdot \cos 3x$
 $\log y = \log \cos x + \log \cos 2x + \log \cos 3x$
 $\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = -\frac{\sin x}{\cos x} - 2 \frac{\sin 2x}{\cos 2x} - 3 \frac{\sin 3x}{\cos 3x}$
 $\Rightarrow \frac{dy}{dx} = -\cos x \cdot \cos 2x \cdot \cos 3x \left[\tan x + 2 \tan 2x + 3 \tan 3x \right]$
- 7) $f(x)$ is continuous on $[-4, 2]$ & differentiable on $(-4, 2)$.
Also, $f(-4) = f(2) = 0$. There exist $c \in (-4, 2)$ such that
 $f'(c) = 0 \Rightarrow 2c+2=0 \Rightarrow 2c=-2 \Rightarrow c = -1 \in \underline{(-4, 2)}$
- 8) $a * e = a + e + 10 = a \Rightarrow e = -10 \notin N$.
Hence no identity element.
- 9) $(x+1)^2 = 4x \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x-1)^2 = 0 \Rightarrow x = 1 \& \underline{\underline{y=2}}$
- 10) $2x + x \cdot \frac{dy}{dx} + y + 2y \cdot \frac{dy}{dx} = 0$
 $\Rightarrow \frac{dy}{dx}(x+2y) = -(y+2x) \Rightarrow \frac{dy}{dx} = \frac{-(y+2x)}{x+2y} //$
- 11) $\tan^{-1} \left(\frac{\cos^2 x/2 - \sin^2 x/2}{\cos^2 x/2 + \sin^2 x/2 - 2 \sin x/2 \cos x/2} \right)$
 $= \tan^{-1} \left[\frac{(\cos x/2 + \sin x/2)(\cos x/2 - \sin x/2)}{(\cos x/2 - \sin x/2)^2} \right]$
 $= \tan^{-1} \left(\frac{1 + \tan x/2}{1 - \tan x/2} \right) = \tan^{-1} \left(\tan\left(\frac{\pi}{4} + \frac{x}{2}\right) \right) = \underline{\underline{\frac{\pi}{4} + \frac{x}{2}}}$

$$12) \underline{\text{LHD}} : - \lim_{h \rightarrow 0} \frac{[3(2-h)^2 + 5] - f(2)}{-h} = \lim_{h \rightarrow 0} \frac{[3(3h-12)]}{-h} = \underline{12}$$

$$\underline{\text{RHD}} : - \lim_{h \rightarrow 0} \frac{[3(2+h)^2 + 5] - f(2)}{h} = \lim_{h \rightarrow 0} \frac{[3(3h+12)]}{h} = \underline{12}$$

$\therefore f(x)$ is differentiable at $x=2$.

SECTION-C

$$13) y = (\log x)^x + x e^{\log x}$$

$$\text{Let } u = (\log x)^x$$

$$\log u = x \log(\log x)$$

$$\frac{1}{u} \cdot \frac{du}{dx} = \frac{x}{\log x} \cdot \frac{1}{x} + \log(\log x)$$

$$\frac{du}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right]$$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = (\log x)^x \left[\frac{1}{\log x} + \log(\log x) \right] + x^{\log x} \cdot \left[\frac{2 \log x}{x} \right]$$

$$14) f \circ g(x) = f(g(x)) = f\left(\frac{x}{x-1}\right) = \left(\frac{x}{x-1}\right)^2 + 2 = \frac{3x^2 - 4x + 2}{(x-1)^2}$$

$$g \circ f(x) = g(f(x)) = g(x^2 + 2) = \frac{x^2 + 2}{x^2 + 2 - 1} = \frac{x^2 + 2}{x^2 + 1}$$

$$f \circ g(2) = \frac{3(2)^2 - 4(2) + 2}{(2-1)^2} = \underline{6} \quad g \circ f(-3) = \frac{(-3)^2 + 2}{(-3)^2 + 1} = \frac{11}{10} //$$

$$15) f(x) = \frac{x-3}{x+1}$$

For one-one :- Let $x_1, x_2 \in A$

$$f(x_1) = f(x_2) \Rightarrow \frac{x_1-3}{x_1+1} = \frac{x_2-3}{x_2+1} \Rightarrow x_1 x_2 + x_1 - 3 x_2 - 3 = x_1 x_2 - 3 x_1 + x_2 - 3$$

$$\Rightarrow 4x_1 = 4x_2 \Rightarrow x_1 = x_2$$

$\therefore f$ is one-one.

For onto :- Let for $y \in B$, there exist $x \in A$ such that

$$y = \frac{x-3}{x+1}$$

$$\Rightarrow xy + y = x - 3 \Rightarrow x(y-1) = -3 - y$$

$$\Rightarrow x = \frac{-3-y}{y-1} \in A$$

$\therefore f$ is onto. Hence bijective.

$$16) f(x) = \begin{cases} k \cos x, & x \neq \frac{\pi}{2} \\ 5, & x = \frac{\pi}{2} \end{cases}$$

LHL:-

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{k \cos x}{\pi - 2x} \quad \text{Let } \pi - 2x = y, \text{ AS } x \rightarrow \frac{\pi}{2}, y \rightarrow 0$$

$$\Rightarrow x = \frac{\pi}{2} - \frac{y}{2}$$

$$= \lim_{y \rightarrow 0} \frac{k \cos\left(\frac{\pi}{2} - \frac{y}{2}\right)}{y} = \lim_{y \rightarrow 0} \frac{k \sin \frac{y}{2}}{\frac{y}{2} \times 2} = \frac{k}{2}$$

RHL:-

$$\lim_{x \rightarrow \frac{\pi}{2}} (5) = 5 \quad \therefore \frac{k}{2} = 5 \Rightarrow \underline{k = 10}$$

(OR)

$$f(x) = \begin{cases} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x}, & \text{if } -1 \leq x < 0 \\ \frac{2x+1}{x-1}, & \text{if } 0 \leq x \leq 1 \end{cases}$$

$$\text{LHL: } \lim_{x \rightarrow 0^-} \frac{\sqrt{1+kx} - \sqrt{1-kx}}{x} \times \frac{\sqrt{1+kx} + \sqrt{1-kx}}{\sqrt{1+kx} + \sqrt{1-kx}} = \frac{RHL:}{\lim_{x \rightarrow 0^+} \frac{2x+1}{x-1}}$$

$$\Rightarrow \lim_{x \rightarrow 0^-} \frac{x+kx - 1+kx}{x(\sqrt{1+kx} + \sqrt{1-kx})} = -1$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \frac{2kx}{x(\sqrt{1+kx} + \sqrt{1-kx})} = -1$$

$$\Rightarrow \frac{2k}{1} = -1 \Rightarrow \underline{k = -1}$$

$$17) y = 3 \cos(\log x) + 4 \sin(\log x)$$

$$y_1 = \frac{-3 \sin(\log x)}{x} + \frac{4 \cos(\log x)}{x}$$

$$xy_1 = -3 \sin(\log x) + 4 \cos(\log x)$$

$$\Rightarrow xy_2 + y_1 = -\frac{3 \cos(\log x)}{x} - \frac{4 \sin(\log x)}{x}$$

$$\Rightarrow x^2 y_2 + xy_1 = -[3 \cos(\log x) + 4 \sin(\log x)]$$

$$\Rightarrow x^2 y_2 + xy_1 = -y \Rightarrow x^2 y_2 + \underline{xy_1} + y = 0$$

(OR)

$$\begin{aligned}
 y &= [\log(x + \sqrt{x^2+1})]^2 \\
 \frac{dy}{dx} &= 2[\log(x + \sqrt{x^2+1})] \cdot \frac{1}{x + \sqrt{x^2+1}} \left[1 + \frac{1}{2\sqrt{x^2+1}} \cdot 2x \right] \\
 &= 2[\log(x + \sqrt{x^2+1})] \cdot \frac{1}{x + \sqrt{x^2+1}} \cdot \left[\frac{\sqrt{x^2+1} + 2x}{\sqrt{x^2+1}} \right] \\
 &= \frac{2[\log(x + \sqrt{x^2+1})]}{\sqrt{x^2+1}}
 \end{aligned}$$

$$\Rightarrow \sqrt{x^2+1} \cdot \frac{dy}{dx} = 2[\log(x + \sqrt{x^2+1})]$$

Squaring,

$$(x^2+1)(\frac{dy}{dx})^2 = 4[\log(x + \sqrt{x^2+1})]$$

$$(x^2+1)(\frac{dy}{dx})^2 = 4y$$

$$\Rightarrow (x^2+1) \cdot 2 \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} + 2x \cdot (\frac{dy}{dx})^2 = 4 \frac{dy}{dx} \quad (\div 2 \frac{dy}{dx})$$

$$\Rightarrow (x^2+1) \frac{d^2y}{dx^2} + x \cdot \frac{dy}{dx} - 2 = 0$$

$$18) \cot^{-1} \frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}$$

$$= \cot^{-1} \frac{\sqrt{(\cos \frac{x}{2} + \sin \frac{x}{2})^2 + (\cos \frac{x}{2} - \sin \frac{x}{2})^2}}{\sqrt{(\cos \frac{x}{2} + \sin \frac{x}{2})^2 - (\cos \frac{x}{2} - \sin \frac{x}{2})^2}}$$

$$= \cot^{-1} \left(\frac{2 \cos x/2}{2 \sin x/2} \right) = \cot^{-1}(\cot x/2) = \frac{x}{2},$$

19) Slope of tangent to the given curve

$$\text{is } \frac{dy}{dx} = \frac{2}{(x-3)^2}$$

But slope given is 2.

$$\therefore \frac{2}{(x-3)^2} = 2 \Rightarrow (x-3)^2 = 1 \Rightarrow x-3 = \pm 1 \Rightarrow x=2, 4 \\ \therefore y=2, -2$$

Eqn of tangent through (2, 2) is $y-2x+2=0$

& Eqn of tangent through (4, -2) is $y-2x+10=0$.

(OR)

$$x^{2/3} + y^{2/3} = 2$$

Diff. w.r.t x ,

$$\frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3} \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/3}$$

$$\Rightarrow \text{Slope of tangent at } (1,1) = -\frac{1}{1} = -1$$

$$\therefore \text{Eqn of tangent is } y-1 = -1(x-1) \Rightarrow y+x-2=0$$

$$\text{Now, slope of normal at } (1,1) = 1$$

$$\therefore \text{Eqn of normal is } y-1 = 1(x-1) \Rightarrow y-x=0$$

20) $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1} \left[\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x-2} \right) \left(\frac{x+1}{x+2} \right)} \right] = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left[\frac{x^2 + x - 2 + x^2 - x - 2}{x^2 - 4 - x^2 + 4} \right] = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1} \left[\frac{2x^2 - 4}{-3} \right] = \frac{\pi}{4} \Rightarrow \frac{2x^2 - 4}{-3} = \tan \frac{\pi}{4} \Rightarrow \frac{4 - 2x^2}{3} = 1$$

$$\Rightarrow 4 - 2x^2 = 3 \Rightarrow 2x^2 = 1 \Rightarrow x = \pm \frac{1}{\sqrt{2}}$$

21) $x = a(\cos t + t \sin t)$

$$\frac{dx}{dt} = a(-\sin t + t \cos t + \sin t) = a \underline{\cos t}$$

$$y = a(\sin t - t \cos t)$$

$$\frac{dy}{dt} = a(\cos t + t \sin t - \cos t) = a \underline{\sin t}$$

$$\frac{dy}{dx} = \frac{a \sin t}{a \cos t} = \tan t$$

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d}{dx}(\tan t) = \sec^2 t \cdot \frac{dt}{dx} = \sec^2 t \cdot \frac{1}{a \cos t} \\ &= \frac{1}{a \cos^3 t} \end{aligned}$$

$$22) \cos^{-1} \frac{x}{a} + \cos^{-1} \frac{y}{b} = \alpha$$

$$\text{Let } \cos^{-1} \left(\frac{x}{a} \right) = \theta \Rightarrow \frac{x}{a} = \cos \theta$$

$$\text{Let } \cos^{-1} \left(\frac{y}{b} \right) = \phi \Rightarrow \frac{y}{b} = \cos \phi$$

$$\text{Now, } \cos(\theta + \phi) = \cos \alpha$$

$$\Rightarrow \cos \theta \cdot \cos \phi - \sin \theta \sin \phi = \cos \alpha$$

$$\Rightarrow \frac{x}{a} \cdot \frac{y}{b} - \sqrt{1 - \frac{x^2}{a^2}} \cdot \sqrt{1 - \frac{y^2}{b^2}} = \cos \alpha$$

$$\Rightarrow \frac{xy}{ab} - \cos \alpha = \sqrt{1 - \frac{x^2}{a^2}} \cdot \sqrt{1 - \frac{y^2}{b^2}}$$

Squaring,

$$\frac{x^2 y^2}{a^2 b^2} + \cos^2 \alpha - \frac{2xy \cos \alpha}{ab} = \left(1 - \frac{x^2}{a^2}\right) \left(1 - \frac{y^2}{b^2}\right)$$

$$\Rightarrow \frac{x^2 y^2}{a^2 b^2} + \cos^2 \alpha - \frac{2xy \cos \alpha}{ab} = 1 - \frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2 y^2}{a^2 b^2}$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy \cos \alpha}{ab} = 1 - \cos^2 \alpha$$

$$\Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy \cos \alpha}{ab} = \underline{\underline{\sin^2 \alpha}}$$

$$23) \text{ Given, } \frac{dx}{dt} = -3 \text{ cm/min}, \frac{dy}{dt} = 2 \text{ cm/min}$$

$$(i) \text{ Perimeter, } P = 2(x+y)$$

$$\Rightarrow \frac{dP}{dt} = 2 \left(\frac{dx}{dt} + \frac{dy}{dt} \right) = 2(-3+2) = \underline{\underline{-2 \text{ cm/min}}}$$

\therefore Perimeter decreases at the rate of 2 cm/min

$$(ii) \text{ Area} = xy$$

$$\frac{dA}{dt} = x \cdot \frac{dy}{dt} + y \cdot \frac{dx}{dt}$$

$$\text{when } x = 10, y = 6,$$

$$\frac{dA}{dt} = 10(2) + 6(-3) = 20 - 18 = \underline{\underline{2 \text{ cm/min}}}$$

\therefore Area increases at the rate of 2 cm/min .

SECTION-D

24) $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}$

$$\log x = 3 \log(\sin t) - \frac{1}{2} \log(\cos 2t)$$

Diff. w.r.t x ,

$$\frac{1}{x} \cdot \frac{dx}{dt} = \frac{3}{\sin t} (\cos t) - \frac{1}{2 \cos 2t} (-2 \sin 2t)$$

$$\Rightarrow \frac{dx}{dt} = \frac{\sin^3 t}{\sqrt{\cos 2t}} (3 \cot t + \tan 2t)$$

Now, $y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

$$\log y = 3 \log(\cos t) - \frac{1}{2} \log(\cos 2t)$$

Diff. w.r.t x ,

$$\frac{1}{y} \cdot \frac{dy}{dt} = \frac{3}{\cos t} (-\sin t) + \frac{1}{2 \cos 2t} \cdot 2 \sin 2t$$

$$\Rightarrow \frac{dy}{dt} = \frac{\cos^3 t}{\sqrt{\cos 2t}} (-3 \tan t + \tan 2t)$$

$$\frac{dy}{dx} = \frac{\cos^3 t}{\sqrt{\cos 2t}} (-3 \tan t + \tan 2t)$$

$$\frac{\sin^3 t}{\sqrt{\cos 2t}} (3 \cot t + \tan 2t)$$

$$= \cot^3 t \left(\frac{-3 \tan t + \tan 2t}{3 \cot t + \tan 2t} \right)$$

$$= \cot^3 t \cdot \left[\frac{\frac{2 \tan t}{1 - \tan^2 t} - 3 \tan t}{3 + \tan t \cdot \frac{2 \tan t}{1 - \tan^2 t}} \right], \tan t$$

$$= \cot^3 t \left(\frac{3 \tan^3 t - \tan t}{3 - \tan^2 t} \right) = -\underline{\underline{\cot 3t}}$$

(OR)

$$(x-a)^2 + (y-b)^2 = c^2$$

Diff. w.r.t x ,

$$2(x-a) + 2(y-b) \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{(x-a)}{y-b} \rightarrow ①$$

Diff. w.r.t x again,

$$\frac{d^2y}{dx^2} = - \left[\frac{(y-b) - (x-a) \cdot \frac{dy}{dx}}{(y-b)^2} \right]$$

$$= - \left\{ \frac{(y-b) - (x-a) \cdot \left[-\frac{(x-a)}{(y-b)} \right]}{(y-b)^2} \right\}$$

$$\therefore \frac{d^2y}{dx^2} = - \left[\frac{(y-b)^2 + (x-a)^2}{(y-b)^3} \right] \rightarrow ②$$

$$\begin{aligned} \text{From } ① \text{ & } ②, \quad & \left[\frac{1 + \left(\frac{dy}{dx} \right)^2}{\frac{d^2y}{dx^2}} \right]^{3/2} = \frac{\left[1 + \left(\frac{x-a}{y-b} \right)^2 \right]^{3/2}}{\left[\frac{(y-b)^2 + (x-a)^2}{(y-b)^3} \right]} \\ & = \frac{\left[\frac{(y-b)^2 + (x-a)^2}{(y-b)^2} \right]^{3/2}}{\left[\frac{(y-b)^2 + (x-a)^2}{(y-b)^3} \right]} \\ & = \frac{-c^3}{-(y-b)^3} \times \frac{(y-b)^3}{c} = \underline{\underline{-c^2}} \end{aligned}$$

$$25) \quad f(x) = 3x^4 - 4x^3 - 12x^2 + 5$$

$$f'(x) = 12x^3 - 12x^2 - 24x$$

For critical points, $f'(x) = 0 \Rightarrow 12x^3 - 12x^2 - 24x = 0$

$$\Rightarrow x(x^2 - x - 2) = 0$$

$$\Rightarrow x(x+1)(x-2) = 0$$

$$\Rightarrow x = 0, -1, 2$$

Intervals	Sign of $f'(x)$	Nature of $f(x)$
$(-\infty, -1)$	< 0	\downarrow
$(-1, 0)$	> 0	\uparrow
$(0, 2)$	< 0	\downarrow
$(2, \infty)$	> 0	\uparrow

$\therefore f(x)$ is st. increasing
in $(-1, 0) \cup (2, \infty)$
& st. decreasing in
 $(-\infty, -1) \cup (0, 2)$.

$$26) x * y = x + y + xy$$

(i) For all $x, y \in R$, $x + y + xy \in R \therefore *$ is binary.

$$(ii) x * y = x + y + xy$$

$$= y + x + xy$$

$= y * x \therefore *$ operation is commutative.

(iii) For $x, y, z \in R$,

$$(x * y) * z = (x + y + xy) * z$$

$$= x + y + xy + z + (x + y + xy)z$$

$$= x + y + z + xy + xz + yz + xyz$$

$$x * (y * z) = x * (y + z + yz)$$

$$= x + y + z + yz + x(y + z + yz)$$

$$= x + y + z + yz + xy + xz + xyz$$

$\therefore *$ operation is associative.

$$(iv) x * e = e * x = x \text{ for all } x \in R$$

$$x + e + xe = x \Rightarrow e(1+x) = 0 \Rightarrow e = 0 \in R - \{1\}$$

$$(v) x * y = e \Rightarrow x + y + xy = 0 \Rightarrow y(1+x) = -x$$

$$\Rightarrow y = \frac{-x}{1+x} \in R - \{-1\}$$

$$(vi) 5 * 3 = 5 + 3 + 5 \cdot 3 = 8 + 15 = \underline{\underline{23}}$$

(OR)

$$A = \{1, 2, 3, 4, 5\}, R = \{(a, b) : |a-b| \text{ is even}\}$$

(i) Let $(a, a) \in R$, $|a-a| = 0$ is even $\Rightarrow R$ is reflexive.

(ii) Let $a, b \in A$.

$|a-b|$ is even $\Rightarrow |b-a|$ is even. Hence symmetric.

(iii) Let $a, b, c \in A$.

$|a-b|$ is even, $(b, c) \in R \Rightarrow |b-c|$ is even

$|a-b| + |b-c| = |a-b+b-c| = |a-c|$ is even $\Rightarrow (a, c) \in R$

Hence transitive.

$\therefore R$ is equivalence relation.

$$27) x\sqrt{1+y} + y\sqrt{1+x} = 0$$

$$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$$

$$\text{Squaring, } x^2(1+y) = y^2(1+x)$$

$$\begin{aligned} \Rightarrow x^2 + x^2y &= y^2 + xy^2 \\ \Rightarrow x^2 - y^2 &= xy^2 - x^2y \\ \Rightarrow (x+y)(x-y) &= -xy(x-y) \Rightarrow y + xy = -x \\ &\Rightarrow y = \frac{-x}{1+x} \end{aligned}$$

Diff. w.r.t λ

$$\frac{dy}{dx} = \frac{-1}{(1+x)^2} //$$

28) Let y be an arbitrary element of f .

$$\therefore y = 9x^2 + 6x - 5$$

$$\Rightarrow y = (3x+1)^2 - 6 \Rightarrow x = \frac{\sqrt{y+6} - 1}{3}$$

Now define $g : [6, \infty) \rightarrow A$ such that $g(y) = x$

$$gof(x) = g(f(x)) = g[(3x+1)^2 - 6] = x$$

$$f \circ g(y) = f(g(y)) = f\left(\frac{\sqrt{y+6}-1}{3}\right) = y$$

$\therefore f$ is invertible & hence $f^{-1} = g$.

$$29) \quad h = R + x$$

$$z^2 = R^2 - x^2$$



$$V = \frac{1}{3}\pi(R^2 - x^2)(R + x) = \frac{1}{3}\pi(R + x)^2(R - x)$$

$$\frac{dV}{dx} = \frac{1}{3}\pi \left[(R-x)^2(R+x) + (R+x)^2(-1) \right]$$

$$= \frac{1}{3} \pi (R+x) [2R - 2x - R - x]$$

$$= \frac{1}{3}\pi(R+x)(R-3x)$$

for max vol, $\frac{dV}{dx} = 0 \Rightarrow R+x=0 \Rightarrow x = -R$ rejected
 $\& R-3x=0 \Rightarrow x = R/3$

$$\frac{d^2V}{dx^2} = \frac{1}{3}\pi \left[(R+x)(-3) + (R-3x) \right]$$

$$= \frac{1}{3}\pi [-2R - 6x] = -\frac{2}{3}\pi(R + 3x) < 0$$

\therefore vol is max when $x = R/3$

$$V = \frac{1}{3} \pi (R-x)(R+x)^2$$

$$= \frac{1}{3} \pi \times \frac{2R}{3} \times \left(\frac{4R}{3} \right)^2 = \frac{8}{27} \left(\frac{4}{3} \pi R^3 \right)$$

(OR)

Let ' x ' be the length of one piece.

$$4a = x \Rightarrow a = \frac{x}{4} \quad 2\pi r = 28 - x \\ \therefore r = \frac{28-x}{2\pi}$$

$$A = a^2 + \pi r^2 \\ = \left(\frac{x}{4}\right)^2 + \pi \left(\frac{28-x}{2\pi}\right)^2 \\ = \frac{x^2}{16} + \cancel{\pi} \frac{(28-x)^2}{4\pi^2}$$

$$\frac{dA}{dx} = \frac{1}{16} \cdot 2x + \frac{1}{4\pi} \cdot 2(28-x) \cdot (-1) \\ = \frac{x}{8} - \frac{28-x}{2\pi}$$

for minimum area, $\frac{dA}{dx} = 0$

$$\Rightarrow \frac{x}{8} - \frac{28-x}{2\pi} = 0$$

$$\Rightarrow \frac{x}{8} = \frac{28-x}{2\pi} \Rightarrow \pi x = 112 - 4x \\ \Rightarrow x = \frac{112}{4+\pi}$$

$$\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{1}{2\pi} > 0 \quad \therefore \text{Area is min when } x = \frac{112}{\pi+4}$$

$$28-x = 28 - \frac{112}{4+\pi} = \frac{28\pi}{\pi+4}$$